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Low-frequency phonon absorption by a magnetically quantized 2D electron system

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Abstract. The rate of absorption of phonons from a monochromatic beam by a 2D electron system in a strong magnetic field is calculated in the diffusive regime and it is seen that, for certain phonon frequencies, the variation of absorption with Fermi energy should have two maxima per Landau level with a separation which scales as $\sqrt{B}$. This calculation is extended into the lower-temperature, quantum Hall regime using a scaling expression for the diffusion constant showing that the phonon absorption should vanish as the temperature is lowered in the same manner as the longitudinal conductivity.

1. Introduction

In recent years there has been much success in using the bulk acoustic phonons of a semiconductor crystal to probe the electronic properties of a low dimensional structure embedded therein (at a heterojunction or quantum well for example) [1]. In this paper I wish to consider the absorption of low-energy phonons, from a non-equilibrium pulse, by a 2D electron system (2DES) in a strong magnetic field in the diffusive regime and as the quantum Hall effect begins to appear.

There are two ways in which the electron temperature affects the low energy physics of the 2DES: firstly there is thermal broadening due to the width of the Fermi distribution and secondly there is the temperature dependence of the inelastic scattering rate. The former effect can be gauged by comparing the thermal energy, $k_B T_e$, with the natural energy scales of the problem: the cyclotron energy, $\hbar\omega_c = \hbar eB/m^*$; the Landau level width, Γ, which depends on the degree of disorder in the system (the true, field independent, measure of disorder is the quantity $2\pi l_c^2\Gamma^2$ where $l_c = \sqrt{\hbar/eB}$ is the cyclotron length; hence Γ scales as $\sqrt{B}$); the Coulomb energy, $e^2/4\pi\epsilon l_c$ and of course the phonon energy $\hbar\omega$. Here I will only consider 'dirty' systems in which the effects of disorder mask any strong correlation effects such as Wigner solidification, the fractional quantum Hall effect (but see [2,3]) or composite fermions; the energy scales for these effects are typically a small fraction of the Coulomb energy and hence the Coulomb interaction can be treated in a mean field way.

The effect of inelastic scattering is usually discussed in terms of the Thouless length [4] $L_{in}(T) \sim T^{-p/2}$, where p is an exponent depending on effective dimensionality and dominant scattering mechanism. This is to be compared with the other important length scales of the system, the cyclotron length, l_c, and the localization length for the quasiparticles at the Fermi energy, $\xi(\mu)$ (here the chemical potential is measured from the centre of the Landau band, $\mu = E_F - (n + \frac{1}{2})\hbar\omega_c$). In this paper I will only consider situations where $k_B T_e$ can be neglected so that there is no thermal broadening (this greatly simplifies the calculations and should not lead to qualitative errors). It will further be assumed that

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$\omega < \Gamma \ll \hbar\omega_c$ so that I will only be considering intra-Landau level processes. When $L_{in}(T_e) < \xi(\mu)$ the electronic motion is diffusive and can be described by simple mean field theories like the self-consistent Born approximation (SCBA) [5] (see [6] for the notation used here). When $L_{in}(T_e) > \xi(\mu)$ Anderson localization [7] is important and the simple mean field picture is inadequate. In this regime the integer quantum Hall effect begins to appear (it is fully developed when $L_{in} \gg \xi(\mu)$). To describe the physics of this regime, renormalization group ideas have been used by Pruisken [8] to derive scaling expressions for the conductivity tensor. The relevant scales are therefore Γ, $e^2/4\pi\epsilon l_c$, $L_{in}(T_e)$, $\xi(\mu)$ and l_c. To simplify matters I shall work in natural units in which $\hbar = \Gamma = l_c = 1$.

The organization of the rest of this paper is as follows. In section 2 the arrangement of the idealized experiment is outlined and an expression for the phonon absorption rate is derived from Fermi's golden rule. In section 3 phonon absorption by a 2DES in the diffusive regime will be discussed. The phonon absorption rate is evaluated in the SCBA using the random phase approximation (RPA) for the screening of the electron–phonon interaction. In section 4 the regime in which the quantum Hall effect begins to appear will be discussed using scaling arguments in order to understand the way in which the phonon absorption decreases as the electron temperature is lowered. Section 5 is a brief summary commenting on possible experimental implications of these results.

2. The model system

The model system to be considered here consists of a 2D layer of electrons embedded in an elastic medium at a temperature T_e. It will be assumed that monochromatic phonons can be injected into the medium in a pulse which travels ballistically to the 2DES. The phonons are labelled by a polarization index, λ, and a 3D wavevector, $\underline{Q}$; the frequency of a phonon in a given mode being $\omega_\lambda(\underline{Q})$. The isotropic Debye approximation will be employed so that $\omega_\lambda(\underline{Q}) = c_\lambda Q$ where c_λ is the relevant speed of sound. It will be convenient to specify a 3D wavevector, $\underline{Q}$, in terms of its projection onto the plane of the 2DES, q, and an angle of incidence, θ, defined such that $\theta = 0$ corresponds to phonons travelling normal to the 2DES. In this notation the Debye approximation has the form $\omega_\lambda(q, \theta) = \tilde{c}_\lambda(\theta)q$ where $\tilde{c}_\lambda(\theta) = c_\lambda/\sin\theta$ is the 2D projection of the sound velocity. Density modulations in the 2DES are labelled by a 2D wave vector q. In natural units $\tilde{c}_\lambda$ is the ratio of the 2D projected speed of sound to the characteristic velocity $v_e = \Gamma l_c/\hbar$. It is therefore a rather small number of the order of 0.1 (in conventional units $c \sim 3 \times 10^3$ m s^{-1} whereas $v_e \sim 10^4$ m s^{-1}).

The electron–phonon coupling is

$$H_{e\phi} = \sum_{\lambda,\underline{Q}} M_\lambda(\underline{Q})\hat{\rho}_q \left(\hat{a}_\lambda(\underline{Q}) + \hat{a}_\lambda^\dagger(-\underline{Q})\right) \tag{1}$$

where $\hat{\rho}_q$ is the electron density operator, $\hat{a}_\lambda(\underline{Q})$ is a phonon annihilation operator and $M_\lambda(\underline{Q})$ is a matrix element which can be written as

$$M_\lambda(\underline{Q}) = \frac{1}{\sqrt{\omega_\lambda(\underline{Q})}}(\beta_0 - i\Xi_0 Q) \tag{2}$$

where β_0 is an effective piezomodulus and Ξ_0 is an effective deformation potential (angular averages have been taken and the finite extent of the electronic wave functions in the

z direction neglected, a more complete treatment would be straightforward and give qualitatively similar results).

The rate at which phonons of a given mode are absorbed by the 2DES is given by Fermi's golden rule as

$$\tau_\lambda^{-1}(\underline{Q}) = 2\pi \sum_m |M_\lambda(\underline{Q})|^2 |\langle m|\hat{\rho}_q|0\rangle|^2 \delta(E_m - E_0 - \omega_\lambda(\underline{Q})) \tag{3}$$

where $|m\rangle$ is the mth eigenstate of the electronic system, $m = 0$ being the ground state, and $\underline{Q} = (q, q_z)$. This can be rewitten in terms of Heisenberg operators for the electrons as

$$\tau_\lambda^{-1}(\underline{Q}) = |M_\lambda(\underline{Q})|^2 \int \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega_\lambda(\underline{Q})t} \langle 0|\hat{\rho}_q(t)\hat{\rho}_{-q}(0)|0\rangle \tag{4}$$

the integral is easily recognized as the dynamic structure factor [9] $S(q, \omega)$ so that

$$\tau_\lambda^{-1}(\underline{Q}) = |M_\lambda(\underline{Q})|^2 S(q, \omega_\lambda(\underline{Q})). \tag{5}$$

The rate at which energy is absorbed by the 2DES from a monochromatic beam of phonons is then $\dot{U} = \omega_\lambda(\underline{Q})\tau_\lambda^{-1}(\underline{Q})$.

3. The diffusive regime

In the diffusive regime, the SCBA can be used to handle the disorder and the RPA to handle the screening. The dynamic structure factor

$$S(q, \omega) = \int \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t} \langle 0|\hat{\rho}_q(t)\hat{\rho}_{-q}(0)|0\rangle \tag{6}$$

can be written in terms of the (retarded) density response function [9]

$$\chi(q, \omega) = \chi'(q, \omega) + \mathrm{i}\chi''(q, \omega) = \mathrm{i}\int_0^\infty \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t} \langle 0|\left[\hat{\rho}_q(t), \hat{\rho}_{-q}(0)\right]|0\rangle \tag{7}$$

via $S = \chi''/2$. In the RPA the density response function can be written in the form

$$\chi(q, \omega) = \frac{\chi_0(q, \omega)}{\epsilon(q, \omega)} \tag{8}$$

where χ_0 is the density response function for non-interacting electrons,

$$\epsilon(q, \omega) = \epsilon'(q, \omega) + \mathrm{i}\epsilon''(q, \omega) = 1 - V(q)\chi_0(q, \omega) \tag{9}$$

is the dielectric function and $V(q) = \alpha/q$ is the bare (quasi-2D) Coulomb interaction. The constant α is the ratio of the Coulomb energy $e^2/4\pi\epsilon l_c$ to the disorder broadening Γ in conventional units. It is not hard to show that

$$S(q, \omega) = \frac{S_0(q, \omega)}{|\epsilon(q, \omega)|^2} \tag{10}$$

where S_0 is the dynamic structure factor in the absence of screening.

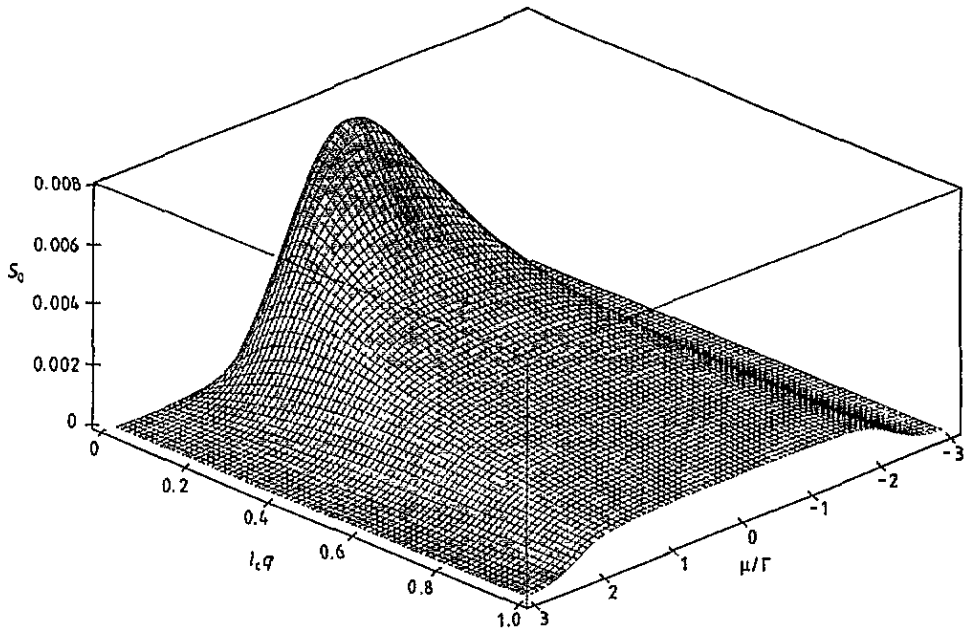

Figure 1. Surface plot showing the dependence of the bare structure factor S_0 on $l_c q$ abd μ/Γ.

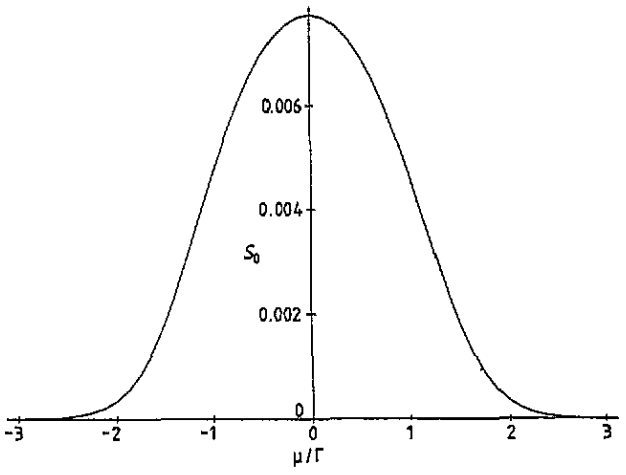

Figure 2. A section through the surface plot in figure 1 at $l_c q = 0.1$.

A consequence of the diffusive motion of the electrons is that, in the hydrodynamic limit, the non-interacting response function must satisfy [10]

$$\chi_0(q, \omega) \sim \frac{1}{2}\rho(\mu)\frac{D(\mu)q^2}{-i\omega + D(\mu)q^2} : \qquad q, \omega \to 0 \tag{11}$$

where $\rho(\mu)$ is the density of states at the Fermi energy. The effective diffusion constant $D(\mu)$ can be calculated in the SCBA and the result is that (in conventional units) $D(\mu) = 4\pi^2 l_c^4 (n + \frac{1}{2})\rho(\mu)/\hbar$ where n is the Landau level index.

Figure 1 shows the variation of S_0 with q and μ assuming that $\tilde{c} = 0.1$. At very small q ($\ll c/D(\mu)$) $S_0 \sim \rho^2 q$, while for large $q (\gg c/D(\mu)) S_0 \sim 1/q$. As might be expected there is a peak in absorption at the Landau band centre which is quite sharp at low q, but broadens out into a low, flat plateau at larger q. Figure 2 shows a section through the surface taken at $l_c q = 0.1$.

The inclusion of the Coulomb screening of the electron–phonon interaction leads to additional structure which can overwhelm the peak in S_0. The response function $\chi(q, \omega)$ does not have a true plasmon pole because of the diffusive nature of the electronic states; it does however have a maximum whenever the real part of $\epsilon(q, \omega)$ vanishes, i.e. where $\chi_0'(q, \omega)V(q) = 1$. This leads to the condition for a maximum in $S(q, \omega)$ to occur that $q = q_s(\mu)$ ($\omega = \tilde{c}q$) where

$$q_s(\mu) = \frac{\alpha\rho(\mu)}{4} \pm \sqrt{\left(\frac{\alpha\rho(\mu)}{4}\right)^2 - \left(\frac{\tilde{c}}{D(\mu)}\right)^2}. \tag{12}$$

Such peaks only occur when the square root is real, i.e. where

$$\rho(\mu)D(\mu) > \frac{4\tilde{c}}{\alpha} \tag{13}$$

but when they do occur their location is well approximated by

$$q_s(\mu) = \frac{\alpha\rho(\mu)}{2}. \tag{14}$$

The value of S at such an absorption resonance is given by

$$S_{\text{peak}}(q_s) = S(q_s, \tilde{c}q_s) = \frac{\alpha^2 D\rho^2}{8\tilde{c}} \tag{15}$$

and, since $\tilde{c}$ is small, these resonances can greatly outweigh the peak due to the maximum in S_0.

Hence we expect that at fixed phonon frequency and angle of incidence the absorption will be a maximum when the chemical potential is at the values $\pm\mu_s(\omega/\tilde{c})$, where $\mu_s(q)$ is the inverse of $q_s(\mu)$ (equation (14)), so there will be two filling factors at which strong absorption takes place. For larger ω the two absorption peaks will be larger and closer together finally merging when $\mu_s(q) = 0$ or, equivalently $q_s(\mu) = \alpha\rho(0)/2$. For larger values of ω there are no absorption resonances. Figure 3 shows a surface plot of S as a function of μ and q; figure 4 is a section through the surface at $l_c q = 0.1$ clearly showing the double-peaked absorption profile.

This effect has been predicted previously by Fromhold *et al* [11] using the Thomas–Fermi argument, but it is seen here in a fully dynamical treatment which includes the diffusive nature of the electronic states.

4. Transition to the quantum Hall regime

At lower temperatures, where the inelastic scattering length becomes greater than the localization length of the states at the Fermi energy, the SCBA argument for the diffusion constant $D(\mu)$ ceases to be valid and this quantity becomes significantly renormalized by

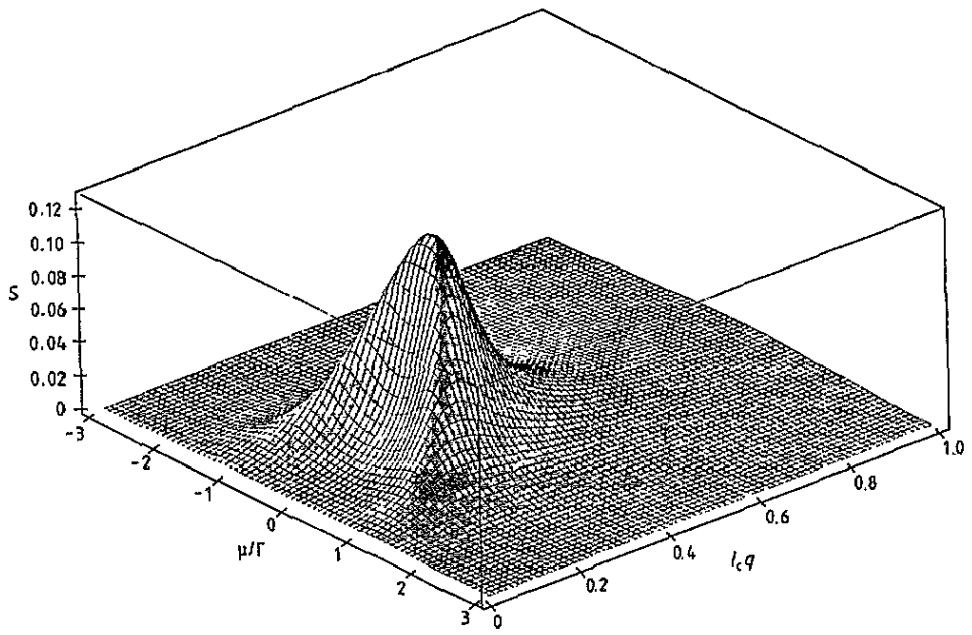

Figure 3. Surface plot showing the dependence of the screened structure factor S on $l_c q$ and μ/Γ.

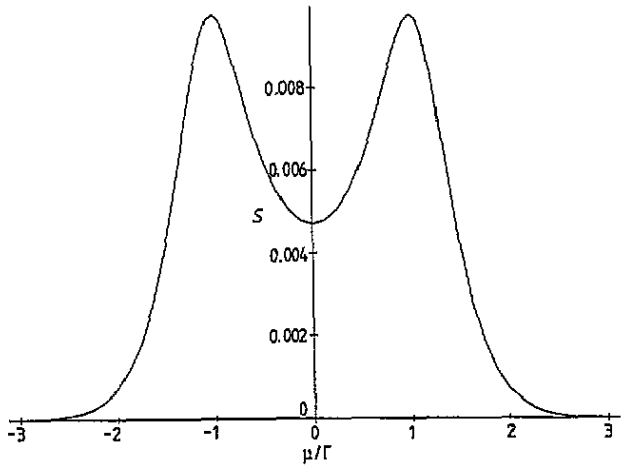

Figure 4. A section through the surface plot in figure 3 at $l_c q = 0.1$.

the effect of Anderson localization. The diffusion constant $D(\mu)$ is expected to satisfy the Einstein relation

$$e^2 D(\mu)\rho(\mu) = \sigma_{xx}(\mu) \tag{16}$$

where σ_{xx} is the longitudinal (dissipative) conductivity (it certainly does in the SCBA). Pruisken [8] has developed a scaling theory for the components of the conductivity tensor close to the quantum Hall fixed point; this predicts that

$$\sigma_{xx}(T_e, \mu) = \sigma_{xx}^0 \, e^{-L_{in}(T_e)/\xi(\mu)} \tag{17}$$

where the localization length is finite except at the critical point, μ^*, of the Landau band, diverging as $\xi(\mu) = \xi_0|\mu - \mu^*|^{-\nu}$ as the critical point is approached (for symmetric disorder we expect the extended states to lie at the centre of the Landau band so that $\mu^* = 0$). Since $\rho(\mu)$ is non-critical at the metal–insulator transition, assuming that $L_{in} \sim T_e^{-p/2}$, one expects that

$$D = D_0 \exp\left\{-\left(\frac{T_e}{T_0(\mu)}\right)^{-p/2}\right\} \quad T_e \to 0 \tag{18}$$

where $T_0(\mu)$ is a characteristic temperature which diverges as $\mu \to 0$ as $|\mu|^{-\nu}$. In figure 5 the variation of q_s with chemical potential μ is plotted for a range of temperatures ($T_e = 0.1T_0, T_0, 10T_0$). Clearly as the electronic temperature is lowered the peaks move to lower q for fixed μ or, equivalently move to smaller μ for fixed q. When D has been reduced sufficiently that $D(\mu) < 4\tilde{c}/\alpha\rho(\mu)$ the square root in (12) becomes negative and the absorption resonances disappear. Physically this occurs because as the temperature is lowered the range of states which are effectively extended (i.e. have a localization length longer than the Thouless length) decreases so that absorption of phonons is suppressed away from the Landau level centre; the absorption peaks consequently move in towards the centre of the band eventually vanishing in the quantum Hall regime, where there is only the absorption due to the extended state at the centre of the Landau band (the renormalization of D is of course ultimately cut off when L_{in} exceeds the physical size of the 2DES).

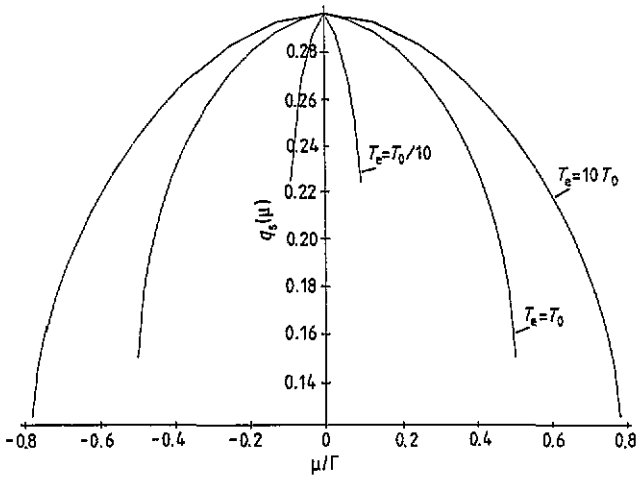

Figure 5. The dependence of location of the absorption resonance in the RPA screened structure factor in the SCBA on μ for $T_e = 0.1T_0, T_0, 10T_0$.

For temperatures sufficiently low that $\omega \gg Dq^2$ the density response function has the form $\chi_0 \sim i\rho Dq^2/2\omega$ while $\epsilon(q, \omega) \to 1$ so that at fixed wave vector the phonon absorption rate should satisfy

$$\ln(\tau^{-1}) \sim \left(\frac{T_0(\mu)}{T_e}\right)^{p/2} T \to 0 \tag{19}$$

hence quantitative measurements of the rate of disappearance of the phonon signal should match the rate at which σ_{xx} tends to zero in transport measurements.

5. Summary

At present, real phonon absorption experiments do not use monochromatic phonons (except in zero field where they can be generated by superconducting tunnel junctions), in fact the phonons are generated by passing a current pulse through a metallic heater which produces a black body spectrum of phonons corresponding to an effective heater temperature which can be calculated from the heater current. The effect of phonon focusing by the lattice [12] means that the phonons from a heater well away from the 2DES will have a relatively narrow spread in angles of incidence.

In section 3 it was shown that in the diffusive regime the phonon absorption rate can be expected to have two maxima as the Fermi level moves through a given Landau level; the weak dependence of the location of these maxima on the phonon angle of incidence means that these peaks could be observed in phonon absorption experiments. This feature could easily be misinterpreted as the effect of spin splitting; the detailed dependence on magnetic field and electron density in a gated device would be needed to distinguish the two effects. The spin splitting at fixed filling factor should be proportional to the (total) magnetic field applied to the 2DES while the effect described above depends on the disorder broadening, which scales as $\sqrt{B}$ (this is true for short-range correlated disorder potentials; if there are long-range correlations in the random potential, due for example to remote ionized donor impurities, then the broadening and hence the position of the absorption peaks are essentially field independent).

In section 4 it was shown that the renormalization of the SCBA results as the temperature is lowered leads to the two phonon absorption peaks moving closer to the centre of the Landau band, eventually disappearing as the quantum Hall effect becomes developed. It was also shown that the rate of decrease of the phonon absorption at a fixed Fermi energy as the temperature was lowered should correspond to the rate of disappearance of the longitudinal conductance in magnetotransport measurements.

Acknowledgments

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